In general, a three-dimensional shock wave-boundary layer interaction can be viewed locally as a two-dimensional one with cross flow and mass transfer[1]—a result of the scavenging vortex in the three-dimensional case. This interpretation is supported by experimental evidence which shows that the extent of a separated flow region is considerably greater for laminar than for turbulent flow for three-dimensional shock-wave/boundary-layer interaction as well as for the two-dimensional case. Thus, in retrospect, it is reasonable to expect a sharp change in the flow separation line as a skewed impinging shock crosses a region of boundary-layer transition. Conversely, a sharp change in an otherwise smoothly curved separation line on a planar surface is most likely indicative of transition because, in the absence of other disturbances in a flow, there is no physical mechanism whereby a shock generator of simple geometry should produce a distorted separation line.

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Technical Comments

Comment on "Lower Bounds on Deformations of Dynamically Loaded Rigid-Plastic Continua"

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Introduction

THE time and displacement bound technique in the early presentation (for example Ref. 1) was developed as a natural extension of the limit analysis theorems for rigid perfectly plastic bodies. While the upper bound principle was subsequently subjected to considerable refinement, no comparable progress was made in the lower bound theorems. A serious drawback in developing the theory further was the idea of kinematically admissible velocity field with stationary in time amplitude taken without alterations from static analysis. As a consequence lower bounds were obtained on the response time rather than on permanent displacements.

A remarkable contribution of Morales and Nevill,[2] is that, considering the amplitude of the velocity field as time variable, they indicated a way of finding lower bounds on displacements.

The objective of this Comment is to clarify some misinterpretations which appeared in Ref. 2 and to outline a correct proof of the new theorem. It should be noted that unlike the general inequality derived in Ref. 2, estimates obtained in all three illustrative examples are correct.

Separable Velocity Field

As a kinematically admissible velocity field $\dot{u}_i^*$, Morales and Nevill[2] took a one-degree-of-freedom velocity field

$$\dot{u}_i^*(x_i, t) = U_i^*(x_i)\dot{T}(t) \tag{1}$$

where the mode function $U_i^*(x_i)$ satisfies kinematic boundary conditions while $\dot{T}(t)$ is a time-variable amplitude. For infinitesimal deformations the strain rate field $\dot{\varepsilon}_{ij}^*$ and dissipation function $D(\dot{\varepsilon}_{ij}^*)$ [resulting from Eq. (1)] are also of the separable form.

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The first remark is that Eq. 1 should be substituted to both sides of the general inequality (8) of Ref. 2, yielding

$$\int_V D(E_{ij}^*)\,dV \int_0^{t_f} \dot{T}(t)\,dt \geqq \int_V (-\rho U_i^*)\left[\int_0^{t_f} \ddot{u}_i\,\dot{T}(t)\,dt\right] dV \tag{2}$$

where $E_{ij}^* = \frac{1}{2}(U_{i,j}^* + U_{j,i}^*)$. Morales and Nevill introduced Eq. (1) only to one side of the aforementioned inequality and consequently were involved in unnecessary and lengthly computations.

Depending upon the choice of the function $\dot{T}(t)$ various information about the response of the body can be deduced. Assuming $\dot{T}(t) = V_o = \text{const}$, Martin[1] obtained from Eq. (2) a simple lower bound on the response time $t_f \leqq t_f^*$ where t_f^* is expressed in terms of U_i^* and initial velocity distribution $\dot{u}_i^o$. Recall that the response time is defined as such time at which actual velocities of all points $x_i \in V$ of the considered body vanish simultaneously

$$\dot{u}_i(x_i, t)\big|_{t=t_f} = 0, \quad u_i^f(x_i) = u_i(x_i, t)\big|_{t=t_f} = \int_0^{t_f} \dot{u}_i\,dt \tag{3}$$

The corresponding value of the displacement is called permanent plastic displacement u_i^f.

The second remark is connected with the proper interpretation of these two definitions. To get lower bounds on permanent displacement a piece-wise linear function $\dot{T}(t)$ was assumed in Eq. (16) of Ref. 2. It is easy to verify, that when this function is substituted into Eq. (2), the lower bound theorem becomes

$$\delta \geqq \tfrac{1}{2} t_f^* \int_V \rho U_i^* \dot{u}_i^0\,dV \Big/ \int_V \rho U_i^* n_i\,dV$$

which now replaces Eq. (30) of Ref. 2.

Bounds on Vector and Scalar Quantities

The third remark relates to Ref. 2 and also to some previous work on that subject.[1] The application of impulsive loading theorems is straightforward in situatons when the displacement vector has only one component. However, the bounding theorems have been formulated for three-dimensional continua and in this case no satisfactory interpretation was given as to what quantity is in fact being bounded. Since formula (2) is of the energy type, one can expect to get from it bounds only on scalar quantities such as response time t_f or an absolute value of the displacement vector $\delta = \max \|u_i^f\|$, where $\|u_i^f\|$ denotes the natural norm of the vector u_i^f. In order to get bounds on δ it is therefore necessary to know a priori the direction cosines of the actual velocity field $n_i(x_i)$. Furthermore the components n_i must be constant during the deformation process. This fact has not been clearly stated in the literature leading to some misinterpretations [for example, in Ref. 2, formula (30) is tensorially incorrect]. Usually n_i are known in problems in which $\dot{u}_i$ has only one component. This situation refers to all illustrative examples so far considered in the literature. This problem has been thoroughly discussed in the author's forthcoming paper.[3]

A Modified Proof

In Ref. 4, a simple lower bound theorem was proved assuming the following form of the time variable amplitude

$$\dot{T}(t) = \max_{x_i \in V} \|\dot{u}_i\| \tag{4}$$

This assumption has a direct physical motivation. First note that the general inequality (8) of Ref. 2 reduces to the equality if $\dot{u}_i = \dot{u}_i^*$. At that instant the left-hand side represents the energy dissipated in the course of plastic deformations while the right-hand side becomes an initial kinetic energy input. The above conclusion does not apply to formula (2) because an approximation, Eq. (1), has already been introduced. However, both sides of the inequality (2) can be brought as close as possible to each other by considering the amplitude $\dot{T}(t)$ to be identical with actual velocity field in the dynamic process at a certain point of the body [formula (4)]. The resulting bound would then be the most exact one.

With the assumption (4), the inequality (2) yields a simple lower bound theorem

$$\delta \geqq \tfrac{1}{2} V_o t_f^* \tag{5}$$

where $V_o = \|\dot{u}_i^o\|$. The preceding formula is valid in the case of uniformly distributed initial velocity. A corresponding result for an arbitrary distribution of initial velocity $\dot{u}_i^o(x_i)$ together with further details of the procedure, can be found in Ref. 4.

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Comment on "Vortices Induced in a Jet by a Subsonic Cross Flow"

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A RECENT article by Durando[1] presented a small perturbation analysis of the flowfield produced some distance downstream of a jet issuing into a cross flow. This method, which is applicable only in regions where the plume makes a small angle to the mainstream, replaces the contra-rotating distributed vortices which are known to dominate the flow in this region by a pair of concentrated vortices connected by a vortex sheet. This analysis is closely akin to the very successful method used initially by Brown and Michael[2] for the flowfield about a slender delta wing with leading-edge separation and with it the author was able to predict the relationship between the vortex strength (Γ) and the distance along its path (ξ) in terms of similarity parameters, viz.

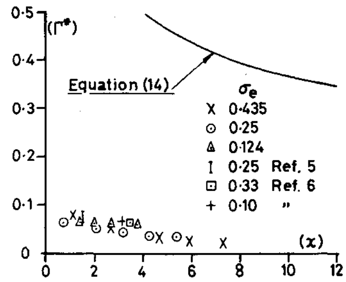

Fig. 1 Comparison between experimentally determined and semiempirical vortex strengths.

$$\Gamma^* = (0.79)/\chi^{1/3} \tag{14}†$$

where $\Gamma^* = \Gamma\sigma_e/4\pi U_\infty d_e$ (13)

and $\chi = \xi\sigma_e/d_e$ (7)

The empirical constant in Eq. (14) was obtained by comparison of the path and separation of the vortices with the experimental data of Pratte and Baines.[3] The model proved to be self consistent since matching the vortex separation to the jet spread led to the correct form for the path. However, as the author states, no comparison could be made between the predicted aand measured vortex strengths due to lack of data.

Comparison with Measured Circulations

An extensive experimental investigation into this problem has been made in the Department of Aeronautics, Imperial College,[4] and careful measurements of both the strength and path of the vortices have been made for a normal jet with velocity ratios (σ_e) of 0.435, 0.25, and 0.124. Calculations of the pressure field induced by these vortices on the wall from which the jet emerges show good quantitative agreement with the measured pressures. Considerable confidence is therefore felt in the accuracy of the vortex strength and path data.

A comparison between the experimental data and Eq. (14) is shown in Fig. (1). A result from Margason and Fearn[5] and two results from Tipping[6] are also indicated. The theoretical values of the vortex strength are an order of magnitude too high even for the region $\chi > 5$, which was quoted as the region for which the analysis might be expected to apply. The experimental values do not correlate on the basis of the similarity variables and there are no indications that there will be any improvement for higher χ values.

Comparison of the Lateral Separation of the Vortices

Because, again, of the lack of data, Durando was obliged to assume that the lateral spacing of the vortices varied in the same way as the jet cross section which had been measured by Pratte and Baines. This conflicts with the Imperial College data which correlates (Fig. 2) downstream of the immediate region of the nozzle in the form

$$(y_o/d_e)\sigma_e^{0.31} = f[(\xi/d_e)\sigma_e^{0.31}]$$

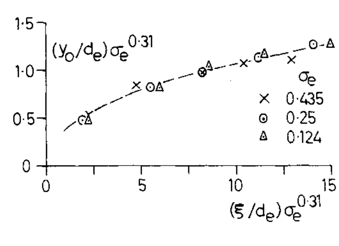

Fig. 2 Correlation of lateral vortex center position with velocity ratio.

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† Equation numbers refer to those appearing in Ref. 1.